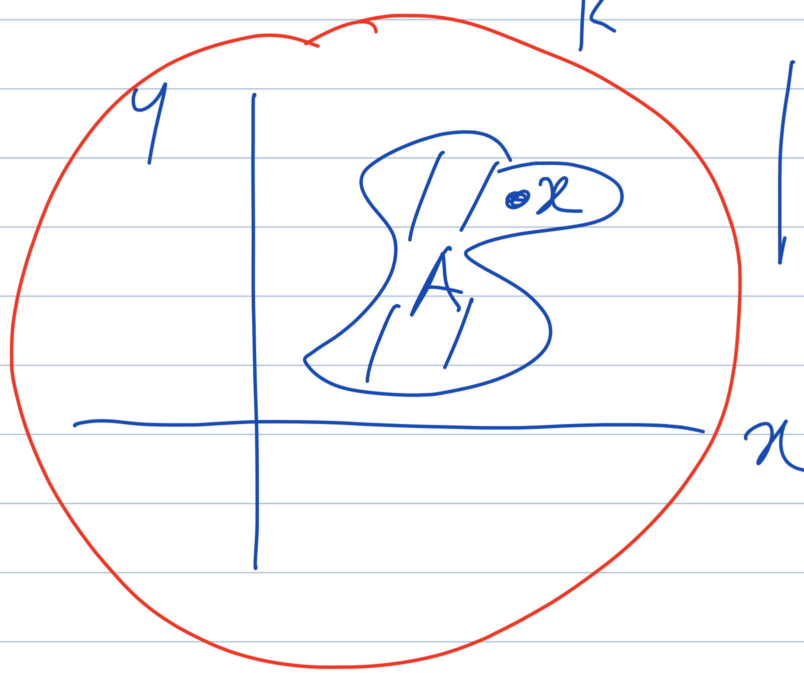
CDI-II · prática · 10/3/21

$A \subset \mathbb{R}^n$ limitado se existe uma bola $B_R(0)$ tal que

$$A \subset B_R(0)$$



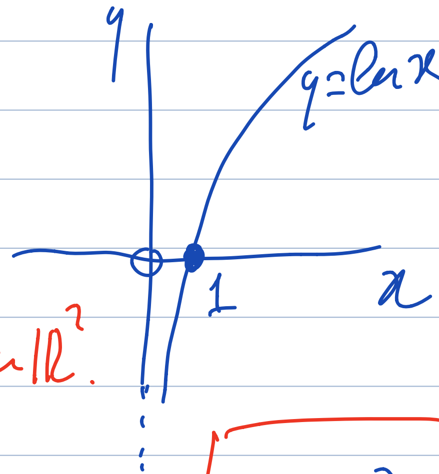
$\boxed{\|x\| < R}$

$A \subset \mathbb{R}^n$ diz-se compacto se for limitado e fechado.

1-5) $\ln(xy) \le 0$

$\iff \boxed{0 < xy \le 1}$

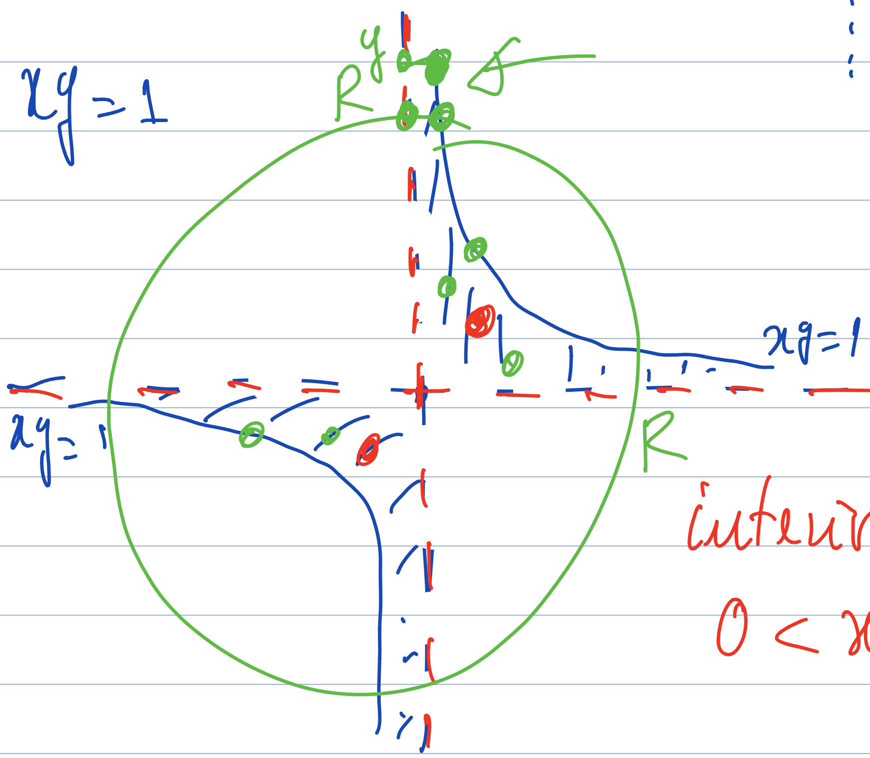
continua pe $\mathbb{R}^2$.



$\boxed{xy = 0}$

$\iff x=0 \vee y=0$

$xy = 1$



interior:

$0 < xy < 1$

$y \ge R \Rightarrow x = \dfrac{1}{R}$

$xy = 1$

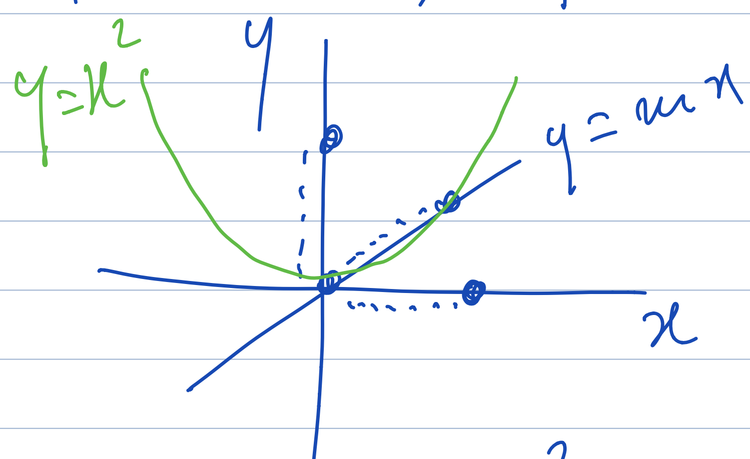
ex $R = 2$

$y = 3 \quad x = \dfrac{1}{3}$.

2-a) $\left|\dfrac{x^2y}{x^2+y^2}\right| = \underbrace{\dfrac{x^2}{x^2+y^2}}_{\le 1}|y| \le |y| \to 0$

2-c) $f(x,y) = \dfrac{x^2y}{x^4+y^2}$

$f(x,0) = 0, \quad f(0,y) = 0$



$m \neq 0$

$f(x,mx) = \dfrac{x^2\,mx}{x^4+m^2x^2} = \dfrac{mx}{x^2+m^2} \to 0$

$$f(x, x^2) = \frac{x^2 x^2}{x^4 + x^4} = \frac{1}{2} \neq 0.$$

$\Rightarrow$ limite não existe.

2-d)

$$\frac{\operatorname{sen} r}{r} \xrightarrow[r \to 0]{} 1$$

Caso notável:

$$\frac{e^r - 1}{r} \xrightarrow[r \to 0]{} 1 \quad ; \quad \frac{\cos r - 1}{r} \xrightarrow[r \to 0]{} 0$$

etc.

$$x \ln x \xrightarrow[x \to 0]{} 0$$

$$\left(\frac{x^2 y}{x^2+y^2}\right)\left(\frac{\operatorname{sen}(x^2+y^2)}{x^2+y^2}\right) \xrightarrow[(x,y)\to(0,0)]{} 0$$

a) notável

— // —

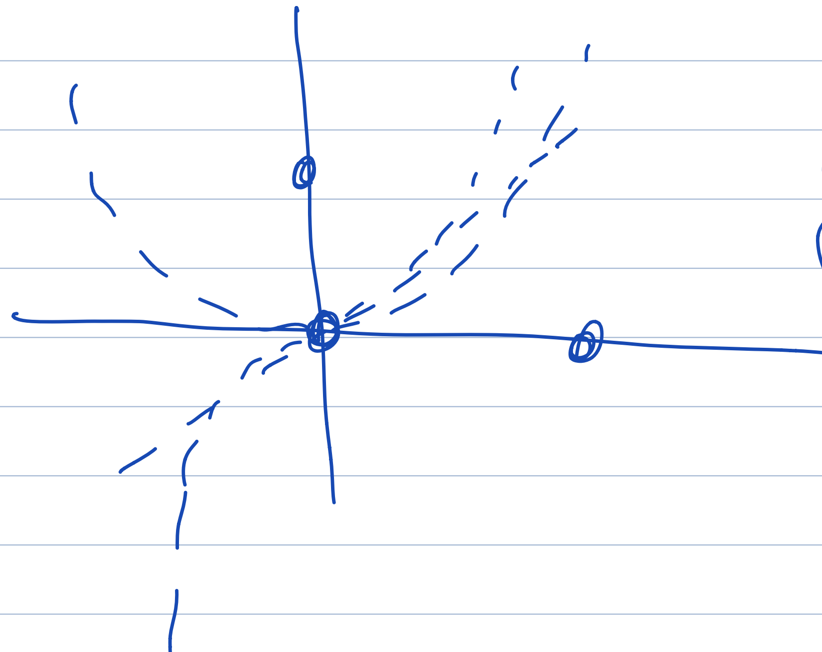
2-e) $f(x,0) = -1$

$f(0,y) = 1$

não existe limite.

— // —

2-f) $x\ln(xy) = \underbrace{x\ln x}_{\text{motáivel}} + \underbrace{x\ln y}_{?}$



$y = x^k$

$k \neq 0$

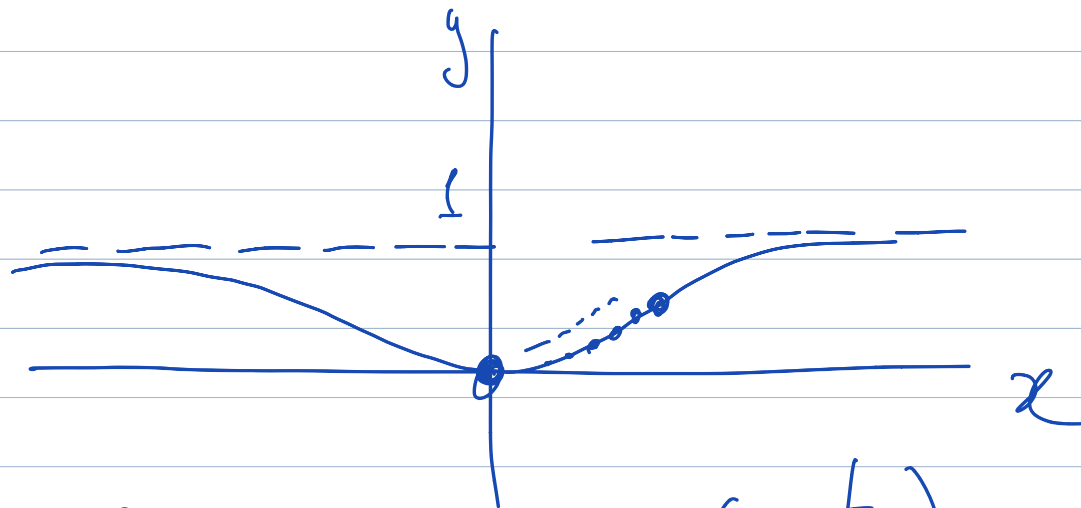
$k > 0$

$x\ln x + x\ln(x^k) =$

$= x\ln x + kx\ln x \longrightarrow 0$

$y = e^{-\frac{1}{x^2}}$

$y = e^{-\frac{1}{x^2}} \quad x \neq 0$



$x \ln x + x \ln\left(e^{-\frac{1}{x^2}}\right) =$

$= x \ln x + x\left(-\frac{1}{x^2}\right)$

$= x \ln x - \frac{1}{x}$

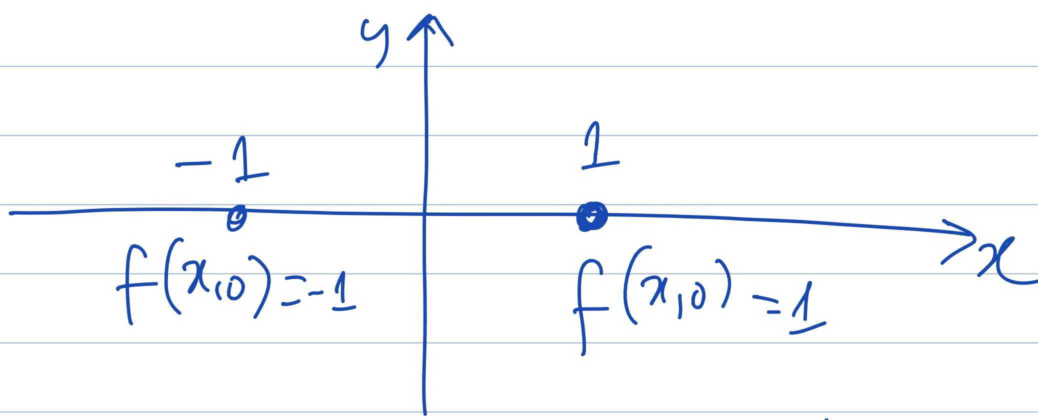
não existe limite.

3 − $\lim_{x \to a} f(x) = f(a)$

(f contínua em a).

3-b) $f(x,0) = \dfrac{x}{\sqrt{x^2}} = \dfrac{x}{|x|}$

$$= \begin{cases} 1 & \text{se } x > 0 \\ -1 & \text{se } x < 0 \end{cases}$$



f contínua em $\mathbb{R}^2 \setminus \{(0,0)\}$

3-d) $\left|\dfrac{x^2}{\sqrt{x^2+y^2}}\right| = \underbrace{\dfrac{|x|}{\|(x,y)\|}}_{\leq 1}|x|$

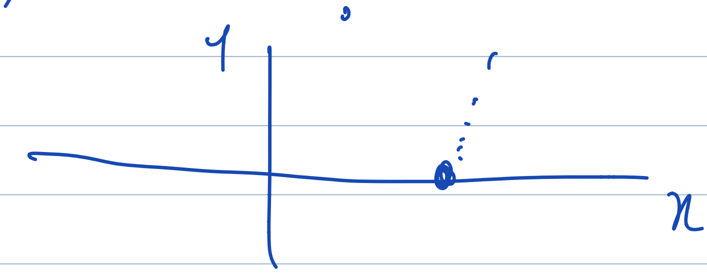
$$|f(x,y)-0| \leq |x| \longrightarrow 0$$

f contínua em $\mathbb{R}^2$.

—— // ——

3-e) $\left|xy^2 \operatorname{sen}\left(\frac{1}{y}\right)\right| \leq |x|\,y^2$

$$\lim_{(x,y)\to(x,0)} f(x,y) = 0 \;!$$



$$f(x,y) = \frac{x\,y}{x^2 + (y-x)^2}$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \,?$$

$$f(0,y) = 0$$

$$f(x,x) = \frac{x^2}{x^2} = 1$$

dois candidatos $\Rightarrow$ não há limite.

$$\frac{x^4+(\operatorname{sen} x)\,y^4}{x^2+x^2y^2+y^4} = \underbrace{\frac{x^4}{x^2+x^2y^2+y^4}} + \underbrace{\frac{(\operatorname{sen} x)\,y^4}{x^2+x^2y^2+y^4}}$$

$$\left|\, x^2\,\underbrace{\frac{x^2}{x^2+x^2y^2+y^4}}_{\leq 1} \right| \leq x^2 \longrightarrow 0$$

$$\left|\, \underbrace{\frac{y^4}{x^2+x^2y^2+y^4}}_{\leq 1}\,\operatorname{sen} x \right| \leq |\operatorname{sen} x| \leq |x|$$

$$|x| \longrightarrow 0$$

✓

$$x^2+x^2y^2+y^4 \geq x^2$$